

# On some Zarankiewicz numbers and bipartite Ramsey numbers for quadrilateral

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(joint work with Janusz Dybizbański and Stanisław Radziszowski)

The Zarankiewicz number  $z(m, n; s, t)$  is the maximum number of edges in a subgraph of  $K_{m,n}$  that does not contain  $K_{s,t}$  as a subgraph. The bipartite Ramsey number  $b(n_1, \dots, n_k)$  is the least positive integer  $b$  such that any coloring of the edges of  $K_{b,b}$  with  $k$  colors will result in a monochromatic copy of  $K_{n_i, n_i}$  in the  $i$ -th color, for some  $i$ ,  $1 \leq i \leq k$ . If  $n_i = m$  for all  $i$ , then we denote this number by  $b_k(m)$ .

In this talk we obtain the exact values of some Zarankiewicz numbers for quadrilateral ( $s = t = 2$ ), and we derive new bounds for diagonal multicolor bipartite Ramsey numbers avoiding quadrilateral. In particular, we prove that  $b_4(2) = 19$ , and establish new general lower and upper bounds on  $b_k(2)$ .

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