Connectivity and other invariants of generalized products of graphs

Susana-Clara López
(joint work with Francesc-Antoni Muntaner-Batle)

An old result of Weichsel establishes that for graphs G and H with at least one edge, the direct product $G \otimes H$ is connected if and only if both G and H are connected and at least one of them is nonbipartite. The lexicographic product of two graphs G and H, with G nontrivial, is connected if and only if G is connected.

Figueroa-Centeno et al. introduced the following product of digraphs in [1]: let D be a digraph and let Γ be a family of digraphs such that $V(F) = V$ for every $F \in \Gamma$. Consider any function $h : E(D) \longrightarrow \Gamma$. Then the product $D \otimes_h \Gamma$ is the digraph with vertex set $V(D) \times V$ and $((a, x), (b, y)) \in E(D \otimes_h \Gamma)$ if and only if $(a, b) \in E(D)$ and $(x, y) \in E(h(a, b))$. The $\otimes_h$-product has been used to establish strong relations among different labelings and specially to produce (super) edge-magic labelings for some families of graphs [3, 5, 6].

In this talk, we introduce the undirected version of the $\otimes_h$-product, which is a generalization of the classical direct product of graphs and, motivated by the $\otimes_h$-product, we also recover a generalization of the classical lexicographic product of graphs, namely the $\circ_h$, that was introduced by Sabidussi en 1961. We study connectivity properties and other invariants in terms of the factors of both, the $\otimes_h$-product and the $\circ_h$-product.

References

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