Computational complexity of counting and phase transitions

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(joint work with Andreas Galainis and Eric Vigoda)

A remarkable connection has been established for 2-spin systems, including the Ising and hard-core models, showing that the computational complexity of approximating the partition function for graphs with maximum degree D undergoes a phase transition that coincides with the statistical physics uniqueness/non-uniqueness phase transition on the infinite D-regular tree (on the uniqueness side: [7] followed by [3, 4], and on the non-uniqueness side: [5] followed by [1, 6]). Despite this clear picture for 2-spin systems, there is little known for multi-spin systems. I will describe the area and discuss recent progress [2] for multi-spin systems: connection to semi-translation invariant Gibbs measures on the infinite D-regular tree, a simple and generic analysis of the second moment for any spin system (using matrix norms), and a connection of the first moment to belief propagation (tree) recursions.

References

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