Rainbow matchings in bipartite graphs and in matroids

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(joint work with Ron Aharoni and Daniel Kotlar)

Let $\mathcal{F} = (F_1, \ldots, F_n)$ be a family of n matchings in a bipartite graph. A (partial) *rainbow matching* in $\mathcal{F}$ is a matching consisting of at most one edge from each F_i. A recent conjecture of Aharoni and Berger [1] asserts that $\mathcal{F}$ has a rainbow matching of size n when each of the n matchings in $\mathcal{F}$ has at least $n+1$ edges. This conjecture generalizes a famous conjecture of Ryser, Brualdi and Stein [4, 6], saying that any Latin square of size n has a transversal of size $n-1$. Aharoni, Charbit and Howard [2] proved that if each F_i has $\lfloor 7n/4 \rfloor$ edges, then $\mathcal{F}$ has a rainbow matching of size n. With Daniel Kotlar we apply a different method to improve this bound to $\lfloor 5n/3 \rfloor$.

A theorem of Woolbright [7], and independently of Brouwer, de Vries and Wieringa [3], asserts that if each F_i has n edges then $\mathcal{F}$ has a rainbow matching of size $n-\sqrt{n}$. A theorem of Drisko [5] asserts that if each F_i has n edges, but $\mathcal{F}$ consists of $2n-1$ matchings, then $\mathcal{F}$ has a rainbow matching of size n. With Ron Aharoni and Daniel Kotlar we prove analogous theorems that guarantee a large rainbow set in the intersection complex of two general matroids, for families of sets in that complex.

References

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