Dense distance magic graphs

Petr Kovář

(joint work with Augustin Žídek)

Let $G = (V, E)$ be a graph on n vertices. A bijection $f : V \to \{1, 2, \ldots, n\}$ is called *distance magic labeling* of G if there exists an integer k such $\sum_{u \in N(v)} f(u) = k$ for all $v \in V$, where $N(v)$ is the set of all vertices adjacent to v. The constant k is the *magic constant* of f and any graph which admits a distance magic labeling is a *distance magic graph*.

The concept of distance magic graphs can be used to schedule fair incomplete tournaments. In a round robin tournament of n players or teams meets each player every other opponent in one game. If each player meets r teams where $r < n - 1$ teams, we naturally expect this incomplete tournament to be equally difficult for each team. This can be modeled using regular distance magic graphs.

In this talk we present a brief overview of results on distance magic graphs and we completely solve the existence of r-regular distance magic graphs if $r \in \{n - 3, n - 5, n - 7, n - 9\}$. Finally, we introduce a couple of possible generalizations that extend the notion of distance magic labelings beyond necessary conditions that limit the existence of regular distance magic graphs.