Combinatorial search problems in vector spaces

Balázs Patkós

(joint work with Tamás Héger and Marcella Takáts)

We consider the following q-analog of the basic combinatorial search problem: let q be a prime power and $\mathrm{GF}(q)$ the finite field of q elements. Let V denote an n-dimensional vector space over $\mathrm{GF}(q)$ and let $\mathbf{v}$ be an unknown 1-dimensional subspace of V. We will be interested in determining the minimum number of queries that is needed to determine $\mathbf{v}$ provided all queries are subspaces of V and the answer to a query U is YES if $\mathbf{v} \leqslant U$ and NO if $\mathbf{v} \nleqslant U$. This number will be denoted by $A(n,q)$ in the adaptive case (when for each queries answers are obtained immediately and later queries might depend on previous answers) and $M(n,q)$ in the non-adaptive case (when all queries must be made in advance).

In the case $n = 3$ we prove $2q-1 = A(3,q) < M(3,q)$ if q is large enough. While for general values of n and q we establish the bounds

$$n \log q \leq A(n,q) \leq (1+o(1))nq$$

and

$$(1-o(1))nq \leq M(n,q) \leq 2nq,$$

provided both n and q tend to infinity.