A large k-tree in connected $K_{1,k+1}$-free graphs

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(joint work with Takeshi Sugiyama)

Let k be an integer with $k \geq 2$. A *k-tree* is a tree whose maximum degree is at most k. We denote by $K_{1,k}$ the star with k vertices of degree 1. A graph G is said to be *$K_{1,k}$-free* if G has no induced subgraph that is isomorphic to $K_{1,k}$.

It is well-known that every connected $K_{1,k}$-free graph has a spanning k-tree, (see [1],) but there are infinitely many connected $K_{1,k+1}$-free graphs that has no spanning k-trees (for example, consider the graph obtained from a large complete graph by attaching $k-1$ new edges to each of the vertices of the complete graph). The purpose of this talk is to find a large k-tree in connected $K_{1,k+1}$-free graphs. Indeed, we show that for every integer k with $k \geq 3$, every connected $K_{1,k+1}$-free graph G has a k-tree T such that $|T| \geq \big(|G|-1\big)^{\beta}+1$, where

$$\beta = \min\big\{ \log_{h(k-1)} \big(h(k-2)+1\big) : h \text{ is a positive integer}\big\}.$$

See Table 1 for the approximate values of β for small k. We will also show that the magnitude of $|G|-1$ (the value of β) in the above result is best possible.

Table 1: The approximate values of β with the values of h attaining it.

k	3	4	5	6	7
β	0.7737 ($h=3$)	0.8842 ($h=4$)	0.9251 ($h=4$)	0.9457 ($h=4$)	0.9579 ($h=5$)

References

[1] B. Jackson, N.C. Wormald, k-walks of graphs, Australas. J. Combin. 2 (1990), 135–146.