Hamiltonicity of claw-free graphs involving neighborhood intersection

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(joint work with Shenggui Zhang)

Let G be a graph and claw be the graph isomorphic to $K_{1,3}$. G is called claw-free if G contains no induced claw. In this paper, we first study the stability of some classes of graphs defined by induced subgraphs with certain neighborhood intersection conditions under the claw-free graph closure proposed by Ryjáček. With these results, it is shown that some sufficient conditions for hamiltonicity in term of forbidden induced subgraphs can be relaxed under the condition of neighborhood intersection. Without the help of claw-free closure, it is also shown that every 2-connected claw-free graph G has a Hamilton cycle if we impose additional condition involving the number of common neighbors of certain pairs of vertices in every induced wounded of G. Our results extend the previous theorems of Bedrossian [1], Broersma and Veldman [2], and Shi [9] on the Hamiltonicity of 2-connected claw-free graphs. Our techniques are closely related to [7] and closure concepts [3, 8].

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