Highly Ramsey-infinite pairs of graphs

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A pair of graphs (H_b, H_r) is highly Ramsey-infinite if there is some constant c such that for large enough n there are at least 2^{cn^2} non-isomorphic graphs on n or fewer vertices that are minimal with respect to the property that when their edges are coloured blue or red, there is necessarily a blue copy of H_b or a red copy of H_r.

We show that a pair of 3-connected graphs is highly Ramsey-infinite if and only if at least one of the graphs in non-bipartite. Further we show that the pair (H_b, H_r) is highly Ramsey infinite for H_r an odd cycle of girth ℓ and H_b any graph with no induced cycle of length ℓ or longer.

In showing the above results, we continue the theory of gadgets called senders and determiners that has been developed over many earlier papers on Ramsey-infinite graphs.