Covering with ordered enclosing for a multiconnected graph

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Let's consider S as a plane; $G=(V,E)$ as a plane graph. Let f_0 be the exterior face of G. For any subset $H\subset S$ define $\mathrm{Int}(H)$ as subset of S, which is union of all the connected components of set $S\backslash H$ not containing exterior facet f_0.

We say that minimal cardinality sequence of edge disjoint trails

$$C^0=v^0e^0_1v^0_1e^0_2...e^0_{k_0}v^0_{k_0},\quad C^1=v^1e^1_1v^1_1e^1_2...e^1_{k_1}v^1_{k_1},\quad \dots,$$
$$C^{n-1}=v^{n-1}e^{n-1}_1v^{n-1}_1e^{n-1}_2\dots e^{n-1}_{k_{n-1}}v^{n-1}_{k_{n-1}}$$

with ordered enclosing such that

$$(\forall m:\ m<n)\quad \left(\bigcup_{l=0}^{m-1}\mathrm{Int}(C^l)\right)\cap\left(\bigcup_{l=m}^{n-1}C^l\right)=\emptyset$$

is Eulerian cover with ordered enclosing for plane graph $G=(V,E)$.

Let's present the algorithm for constructing such a trail for a multiconnected graph. This algorithm applies the concept of nesting value as in earlier papers (for example, in [1]).

Algorithm OptimalMultiComponent

Input: plane graph G.
Output: C^s_j, $j=1,...,|V_{odd}|/2$, covering of graph G by trails with ordered enclosing, $s=1,2,...$ the number of connected component.

Step 1. Define a set X of all connected components for graph G and $\forall x\in X$ define their nesting value $K(x)$.
Step 2. Construct a full abstract graph $\Im$, its vertices be the connected components X of graph G, and edge lengths (weights) are equal to a distance between the nearest vertices of these components.
Step 3. Find minimal spanning tree $T(\Im)$ for $\Im$.
Step 4. Add all edges of this spanning tree to graph G: $G_\Im=G\cup T(\Im)$.
Step 5. Run algorithm **OptimalCover** [1] for graph $G_\Im$.

As for the main idea of algorithm OptimalCover it constructs a covering of a plane graph G by trails with ordered enclosing using the shortest matching of a full graph $K_{|OddV(G)|}$ (its vertices are the vertices of odd degree $OddV(G)\in G$). The additional edges between the trails of a covering are taken as the edges of the shortest matching on $K_{|OddV(G)|}$.

Algorithm OptimalMultiComponent allows to construct covering by polynomial time $O(|V|^3)$.

References

[1] T. Panyukova, E. Savitskiy, Optimization of resources usage for technological support of cutting processes, Proc. CSIT 2010, 66–70.