Replication in critical graphs and square-free monomial ideals

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(joint work with Matěj Stehlík and Riste Škrekovski)

The replication of a set W of vertices of a graph is the operation that adds a copy of each vertex w in W, one at a time, and connects it to w and all its neighbours. Francisco, Hà and Van Tuyl [1] conjectured that for every positive integer k and every k-critical (i.e., critically k-chromatic) graph, there is a set of vertices whose replication produces a $(k+1)$-critical graph. Their motivation comes from commutative algebra, namely from questions about square-free monomial ideals in polynomial rings.

We describe an infinite family of counterexamples to the conjecture which also resolves some of the related algebraic problems (see also [2]). We use this opportunity to give a basic presentation of the algebraic context to a graph theory audience.

References

[1] C.A. Francisco, H.T. Hà, A. Van Tuyl, A conjecture on critical graphs and connections to the persistence of associated primes, Discrete Math. 310 (2010), 2176–2182.

[2] T. Kaiser, M. Stehlík, R. Škrekovski, Replication in critical graphs and the persistence of monomial ideals, arXiv:1301.6983.