Proof of a conjecture of Thomassen on Hamilton cycles in highly connected tournaments

John Lapinskas

(joint work with Daniela Kühn, Deryk Osthus, and Viresh Patel)

A conjecture of Thomassen [4] states that there exists $f : \mathbb{N} \to \mathbb{N}$ such that for every $k \in \mathbb{N}$, every strongly $f(k)$-connected tournament contains k edge-disjoint Hamilton cycles. A classical theorem of Camion, that every strongly connected tournament contains a Hamilton cycle, implies that we may take $f(1) = 1$. So far, even the existence of $f(2)$ was open. In [3], we prove Thomassen's conjecture by showing that we may take $f(k) = O(k^2 \log^2 k)$. This is best possible up to the logarithmic factor. As a tool, we show that every strongly $10^4 k \log k$-connected tournament is k-linked (which improves a previous exponential bound). The proof of the latter is based on a fundamental result of Ajtai, Komlós and Szemerédi [1, 2] on asymptotically optimal sorting networks.

References

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