Hamilton decompositions of graphs and digraphs

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(joint work with Deryk Osthus)

I will discuss several recent results on decompositions of graphs and digraphs into Hamilton cycles and perfect matchings. One example is a conjecture of Kelly from 1968, which states that every regular tournament on n vertices can be decomposed into $(n-1)/2$ edge-disjoint Hamilton cycles. In [5] we proved this conjecture for large n. In fact, we proved a far more general result, based on our recent concept of robust expansion and a new method for decomposing graphs: we showed that every sufficiently large regular digraph G on n vertices whose degree is linear in n and which is a robust outexpander has a decomposition into edge-disjoint Hamilton cycles. (Roughly speaking, a digraph is a robust outexpander if its expansion is resilient to the deletion of a small fraction of vertices or edges.)

This enables us to obtain numerous further results, e.g. as a special case we confirm a conjecture of Erdős on packing Hamilton cycles in random tournaments. We also apply our result to solve a problem on the domination ratio of the Asymmetric Travelling Salesman problem, which was raised e.g. by Glover and Punnen as well as Alon, Gutin and Krivelevich. As a final example, our result is an ingredient in the proofs of the following three conjectures [4, 1, 2, 3]:

- the long-standing 1-factorization conjecture on decompositions of dense regular graphs into perfect matchings,
- a conjecture of Nash-Williams from 1970 on decompositions of dense regular graphs into Hamilton cycles,
- a conjecture of Nash-Williams from 1970 on the number of edge-disjoint Hamilton cycles in graphs of given minimum degree.

The latter three results are joint work with B. Csaba, A. Lo, D. Osthus and A. Treglown.

References

[1] B. Csaba, D. Kühn, A. Lo, D. Osthus, A. Treglown, Proof of the 1-factorization and Hamilton decomposition conjectures II: the bipartite case, preprint.

[2] B. Csaba, D. Kühn, A. Lo, D. Osthus, A. Treglown, Proof of the 1-factorization and Hamilton decomposition conjectures III: approximate decompositions, preprint.

[3] D. Kühn, A. Lo, D. Osthus, Proof of the 1-factorization and Hamilton decomposition conjectures IV: exceptional systems for the two cliques case, preprint.

[4] D. Kühn, A. Lo, D. Osthus, A. Treglown, Proof of the 1-factorization conjecture I: the 2-clique case, preprint.

[5] D. Kühn, D. Osthus, Hamilton decompositions of regular expanders: a proof of Kelly's conjecture for large tournaments, Adv. Math. 237 (2013), 62–146.