2-factors in graphs

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(joint work with Jan van den Heuvel)

A 2-factor in a graph is a set of disjoint cycles covering all vertices of the graph. A complete characterization of the maximal graphs without 2-factors is presented. The proof is based on the general 2-factor theorem. Also an easy proof of the theorem that any $(2r+1)$-regular graph with at most $2r$ bridges has a 2-factor is given, and moreover all $(2r+1)$-regular graphs with $2r+1$ bridges without a 2-factor are found. This generalizes Julius Petersen's famous theorem (1891) that any 3-regular graph with at most two bridges has a 1-factor, and moreover the Sylvester graphs. The results will be put into a historical context. The first to obtain the general 2-factor theorem was Hans Boris Belck in 1949 in his ph.d. thesis, written at the University of Frankfurt when he was only 20 years old. In fact Belck proved the general k-factor theorem, and he also presented the first purely graphtheoretic proof of Tutte's 1-factor theorem. But his terminology was unusual and his proofs not as easy to read as the very elegant theory of 1-factors, presented in 1950 by T. Gallai (who obtained as a byproduct what we now call the Edmonds-Gallai Theorem). In 1951 Tutte proved the general f-factor theorem as a generalization of the k-factor theorem, and in 1953 he discovered how to reduce it to his 1-factor theorem.