How to construct regular map without any exponent

Veronika Hucíková

By a *graph* we will mean any composition of 0-dimensional and 1-dimensional cells, which are *vertices* and *edges* of the graph. So we will allow semi-edges, loops and also multiple edges. A graph embedded in the surface will be called a *map*. We will consider orientable surfaces only.

We will describe a map in language of algebra. We will use the set of darts D for this purpose, where *darts* are in fact semi-edges and two permutations on D called *dart-flip* and *rotation*, labeled L and R, respectively. The permutation L maps every dart onto the dart which forms an edge with it, or onto itself if this dart is in semi-edge. The permutation R maps every dart onto the dart with the same initial vertex, exactly that one which follows given dart on the surface as one moves around this initial vertex in the clockwise orientation of the surface. So the map M can be represented by triples $(D; R, L)$.

In this notation we can also define what is a regular map. An *automorphism* of map $M = (D; R, L)$ is any permutation f on D such that $f(aR) = (fa)R$ and $f(aL) = (fa)L$ for each dart $a \in D$. All automorphisms of the map M form the group $Aut(M)$. Map $M = (D; R, L)$ is *regular* if $Aut(M)$ is regular on D. In a regular map, all vertices have the same degree, say, k, and all faces of the map have the same length, say, m; the map is then said to have *type* (k, m). The types (k, m) such that $1/k + 1/m < 1/2$ are known as *hyperbolic*.

After that we will define an exponent. Let $M = (D; R, L)$ be a map and e be some natural number. We will consider mapping $M = (D; R, L) \mapsto M^e = (D; R^e, L)$. If e is coprime with all degrees of vertices of M, then this mapping doesn't change structure of underlying graph. If $M \cong M^e$ we say that M has an *exponent* e. We would like to construct a regular map of a given type without any exponent. This map can be very big in general, so we will use a technic of covering and the following lema:

Lemma Consider a map $M = (D; R, L)$ where $|D| = n$ and $n > 6$. If $\langle R, L\rangle \cong S_n$ or A_n, then the canonical regular cover $\tilde{M} = (\langle R, L\rangle; R, L)$ has an exponent e if and only if M has an exponent e.