Location of a tree network for a finite set

Anatoly Panyukov

Let's consider problem $\Theta(G, V, b, c, \Phi)$

$$C(\varphi) = \sum_{j \in J} c(j, \varphi(j)) + \sum_{[i,j] \in E} b([i,j], \varphi(i), \varphi(j)) \to \min_{\varphi \in \Phi} \tag{1}$$

for given tree $G = (J, E)$, *finite* set V, functions $b: \ E \times V^2 \to \mathbf{Z}$, $c: \ J \times V \to \mathbf{Z}$, and set Φ of acceptable placement of all nodes $j \in J$ onto items $v \in V$.

This problem $\Theta = \Theta_V$ is known as *Weber problem* for tree network where $\Phi = \Phi_V = \{\varphi: \ J \to V\}$ (i.e. represents all single-valued function). The polynomial algorithm [2] with computational complexity $O(|J||V|^2)$ solves this problem.

Statement of this problem as integer linear programming problem, and demonstration that its relaxation has the integer optimal solution are presented. In this case the existence of the integer saddle point succeeds the proof of decomposition algorithms for problems difference from Θ_V presence of additional constraints.

If for (1) holds

$$\Phi = \Phi_Q = \{\varphi: \ J \to V | \ \forall i, j \in J \, (i \neq j \Rightarrow \varphi(i) \neq \varphi(j))\}$$

(i.e. Φ_Q is set of all injective functions), then problem $\Theta = \Theta_Q$ is *quadratic assignment problem* [1] for tree settable network.

Statement of this problem in the form of integer linear programming problem is presented. It is proved that this problem has integer optimal solution. The used restriction decomposition scheme solves internal problem as Weber problem, and external one as convex programming problem.

References

[1] R. Horst, H. Tuy, Global optimization: Deterministic approaches, Springer, 1993.

[2] A. Panyukov, B. Pelzwerger, Polynomial algorithms to finite Weber problem for a tree network, J. Comput. Appl. Math. 35 (1991), 291–296.